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Exact Solutions in SFT and Marginal Deformation in BCFT

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Abstract: In this note we will study solution of open bosonic string field theory based on action of operators from chiral algebra of boundary conformal field theory on identity element of string field theory star algebra. We will demonstrate that the string field theory action for fluctuation fields around this classical solution can be mapped to the string field theory action defined through the new boundary conformal field theory that arises from the original one through the marginal deformation inserted on the world-sheet boundary.

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1. Introduction

Much work has focused on the search for classical solutions of cubic bosonic open string field theory (SFT) [1] (For review, see [2]–[5].). Despite important technical progress in the understanding of the open string star product-notably the discovery of a new connections with non-commutative field theory [6]–[15] it is still very difficult to find analytic classical solution. Exception is the special form of SFT, vacuum string field theory (VSFT), where exact results have been obtained [18]–[26].

In our recent papers [27]–[31] we found some exact solutions of SFT equation of motion.[1] All these solutions have the property that when we expand the string field around this solution and insert it into the SFT action then the new BRST operator Q' suggests that SFT action for fluctuation modes is in some sense related to the SFT action defined around the new background boundary conformal field theory ($BCFT''$) that arises from the original $BCFT$ by presence of marginal deformation on the world-sheet boundary. The possible relation between these two actions will be indication of the background independence of SFT [53]–[57] which unfortunatelly is not explicitly seen in its formulation. Let us say more about this issue.

This issue arises because SFT can be written after choosing what amounts to a classical solution, namely $BCFT$ with central charge $c = 0$. As a result the SFT actions S_1 , S_2 written using two different $BCFT's$ $BCFT_1$ and $BCFT_2$ are not manifestly equivalent. However it was shown in [53]–[57] that for the case when $BCFT_1$ and $BCFT_2$ are nearby theories related by infinitesimal marginal deformation, one can prove that S_1 and S_2 are the same action expanded around different solutions.

In this paper we will try to study this problem from slightly different point of view. In particular, we focus on the relation between solution of SFT equation of motion given in [27]–[31] and the marginal deformation in $BCFT$. Our goal is to show that when we expand string field around classical solution and insert it into the original SFT action

[1] For closely related papers, see [34]–[52]. For other papers considering exact solutions in SFT, see [38, 39, 40, 41].

S which is defined on background $BCFT$, we obtain after suitable redefinition of the fluctuation modes the SFT action S' defined on $BCFT''$ that is related to the original $BCFT$ by inserting marginal deformation on the boundary of the world-sheet (For more details about marginal deformations in $BCFT$, see [32].). To say differently, we will show that two SFT actions S, S' written using two different $BCFT$, $BCFT''$ which are related by marginal deformation, are in fact two SFT actions expanded around different classical solutions.

In order to show this equivalence will consider operators, that determine exact solution of SFT, which belong to the chiral algebra $\mathcal{W}$ of $BCFT$.[2] More precisely, we will construct the solution of SFT equation of motion that is based on an action of some operator $\mathbf{W}$ from $\mathcal{W}$ on the SFT star algebra identity element $\mathcal{I}$. Then we will study the fluctuation modes around this solution. It turns out that after performing suitable redefinition of the fluctuation modes we will get correlation functions[3] which have the same form as the correlation functions in the deformed $BCFT''$ that arises from the original one by insertion of marginal interaction $\mathbf{W}(x)$ on the world-sheet boundary. In order to be able to perform such a identification we took the operators $\mathbf{W}$ from $\mathcal{W}$ since only in that case we have well defined deformed correlation functions in $BCFT$ containing bulk and boundary operators as well as operator $\mathbf{W}(x)$ [32]. In other words we will show that the SFT action S' for fluctuation modes is the same as the SFT action defined by the second $BCFT''$. This result can be considered as more precise extension of the analysis given in [27] in the sense that we explicitly show the equivalence of these two actions. We mean that this result could be considered as further indication of the background independence in SFT [53]–[57].

The organization of this paper is as follows. In the next section (2) we review the basic facts about SFT, bulk an boundary CFT. Then we turn to the construction of the classical solution of SFT equation of motion following [27]–[31]. In section (3) we will study fluctuation modes above the classical solution and we will find the precise relation between the SFT S' for fluctuation modes and SFT defined on $BCFT''$. In conclusion (4) we will outline our results.

2. General solutions

We begin this section with the review the basic facts about bosonic string field theory, following mainly [2, 3, 4]. Gauge invariant string field theory is described with the full Hilbert space of the first quantized open string including b, c ghost fields subject to the condition that the states must carry ghost number one, where b has ghost number -1, c has ghost number 1 and $\mathrm{SL}(2, \mathbb{C})$ invariant vacuum $|0\rangle$ carries ghost number 0. We denote $\mathcal{H}$ the subspace of the full Hilbert space carrying ghost number 1. Any state in $\mathcal{H}$ will be denoted as $|\Phi\rangle$ and corresponding vertex operator Φ is the vertex operator that creates state $|\Phi\rangle$ out of the vacuum state $|0\rangle$

$$|\Phi\rangle = \Phi\,|0\rangle\,. \tag{2.1}$$

[2]For very nice review to the subject of $BCFT$ see for example [60, 61] and reference therein.

[3]These $BCFT$ correlation functions define SFT action S in the CFT description [48, 49].

Since we are dealing with open string theory, the vertex operators should be put on the boundary of the world-sheet.[4]. The string field theory action is given [1]

$$S = -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\left\langle I\circ\Phi(0)Q\Phi(0)\right\rangle + \frac{1}{3}\left\langle f_1\circ\Phi(0)f_2\circ\Phi(0)f_3\circ\Phi(0)\right\rangle\right), \tag{2.2}$$

where g_0 is open string coupling constant, Q is BRST operator and $<>$ denotes correlation function in the combined matter ghost conformal field theory in the upper half plane $\mathrm{Im}z \geq 0$. I, f_1, f_2, f_3 are conformal mapping exact form of which is reviewed in [2] and $f_i \circ \Phi(0)$ denotes the conformal transformation of $\Phi(0)$ by f_i. For example, for Φ a primary field of dimension h, then $f_i \circ \Phi(0) = (f_i'(0))^h \Phi(f_i(0))$. In our calculation we use convention from the very nice review [2]

$$\begin{aligned}
T_m(z) &= -\frac{1}{\alpha'}\partial_z X_L^{\mu}(z)\partial_z X_L^{\nu}\eta_{\mu\nu}\,, \\
\overline{T}_m(\overline{z}) &= -\frac{1}{\alpha'}\partial_{\overline{z}} X_R^{\mu}(\overline{z})\partial_{\overline{z}} X_R^{\nu}(\overline{z})\eta_{\mu\nu}\,, \\
X_L^{\mu}(z)X_L^{\nu}(w) &\sim -\frac{\alpha'}{2}\eta^{\mu\nu}\ln(z-w)\,, \\
X_R^{\mu}(\overline{z})X_R^{\nu}(\overline{w}) &\sim -\frac{\alpha'}{2}\eta^{\mu\nu}\ln(\overline{z}-\overline{w})\,, \\
X_L^{\mu}(z)X_R^{\nu}(\overline{w}) &\sim -\frac{\alpha'}{2}\eta^{\mu\nu}\ln(z-\overline{w})
\end{aligned} \tag{2.3}$$

with the BRST operator

$$\begin{aligned}
Q &= \frac{1}{2\pi i}\int_C dz j_B(z) - \frac{1}{2\pi i}\int_C d\overline{z}\overline{j}_B(\overline{z})\,, \\
j_B(z) &= c(z)\left[T_m(z) + \frac{1}{2}T_{gh}(z)\right], \\
\overline{j}_B(\overline{z}) &= \overline{c}(\overline{z})\left[\overline{T}_m(\overline{z}) + \frac{1}{2}\overline{T}_{gh}(\overline{z})\right],
\end{aligned} \tag{2.4}$$

where $j_B(z)$ is holomorphic and $\overline{j}_B(\overline{z})$ is anti-holomorphic current and where $T_{ghost}\,, \overline{T}_{ghost}$ are holomorphic and anti-holomorphic stress energy tensor for the ghosts. In what follows we will not need to know the explicit form of the ghost contribution.

It turns out that the natural definition of the string field theory is in language of $BCFT$ [48, 49]. $BCFT$ are usually regarded as associated boundary theories to the bulk CFT theories. Recall that bulk CFT are defined on the whole complex plane and they appear in the world-sheet description of the closed strings. Their state spaces $\mathcal{H}^P$ contain all the closed string modes and the coefficients $C = C^P$ of their operator product expansions (OPE) encode closed string interactions. The space $\mathcal{H}^P$ is equipped with the action of a hamiltonian H^P and of field operators $\phi(z,\overline{z})$. According to state-field correspondence we have an identification

$$\phi(z,\overline{z}) = \Phi^P(|\phi\rangle, z, \overline{z})\,, \qquad \text{for all } |\phi\rangle \in \mathcal{H}^P\,. \tag{2.5}$$

[4]Since these states describe open string they belong to the class so-named boundary operators in $BCFT$ that live on the real line $y = 0$ for $z = x + iy$ [61].

Among the fields of a CFT one distinguishes so-called chiral fields which depend on only one of the coordinates z or $\overline{z}$ so that they are either holomorphic, $W = W(z)$, or anti-holomorphic, $\overline{W} = \overline{W}(\overline{z})$. The (anti)-holomorphic fields of given theory, or their Laurent modes $W_n\,,\overline{W}_n$ defined through

$$W(z) = \sum W_n z^{-n-h}\,, \qquad \overline{W}(\overline{z}) = \sum \overline{W}_n \overline{z}^{-n-\overline{h}}\,, \tag{2.6}$$

generate two commuting chiral algebras $\mathcal{W}$ and $\overline{\mathcal{W}}$. The most important of these chiral fields, the Virasoro fields $T(z), \overline{T}(\overline{z})$ with modes $L_n\,, \overline{L}_n$ are among the chiral fields and numbers $h\,, \overline{h}$ are the (half-)integer conformal weights of $W(z)\,, \overline{W}(\overline{z})$.

$BCFT$ are conformal field theories on the upper half-plane $\mathrm{Im} z \geq 0$ which in the interior $\mathrm{Im} > 0$ are locally equivalent to the given bulk theory: The state space $\mathcal{H}^H$ of the $BCFT$ is equipped with the action of a hamiltonian H^H and of bulk fields

$$\phi(z, \overline{z}) = \Phi^H(|\phi\rangle\,, z, \overline{z})\,, \tag{2.7}$$

which are assigned to the same fields as were used to label filed in the bulk theory. However these fields ϕ act on a different space of states $\mathcal{H}^H$. We also demand that all the leading terms in the OPE's of bulk fields coincide with the OPE's in the bulk theory. Having the same singularities as in the bulk theory means that the boundary conditions do not affect the equation of motion in the bulk. We must also require the boundary theory to be conformal. This is guaranteed if the Virasoro field obeys the following gluing condition

$$T(z) = \overline{T}(\overline{z})\,, \qquad z = \overline{z}\,. \tag{2.8}$$

We also presume that all chiral fields $W(z), \overline{W}(\overline{z})$ can be extended analytically to the real line and that there exist a local automorphism Ω-called the gluing map of the chiral algebra $\mathcal{W}$ such that

$$W(z) = \Omega\left(\overline{W}\right)(\overline{z})\,, \qquad z = \overline{z}\,. \tag{2.9}$$

Now the assumption of the existence of the gluing map Ω has powerful consequence that it gives rise to an action of one chiral algebra $\mathcal{W}$ on the state space $\mathcal{H}^H$ of the boundary theory. More precisely, we combine the chiral fields $W(z)$ and $\Omega\overline{W}(\overline{z})$ into single object $\mathbf{W}(z)$ defined on the whole complex plane such that

$$\mathbf{W}(z) := W(z)\,, \qquad \mathrm{Im}\, z \geq 0\,, \qquad \mathbf{W}(z) := \Omega\overline{W}(\overline{z})\,, \qquad \mathrm{Im}\, z < 0\,. \tag{2.10}$$

Thanks to the gluing condition along the boundary this field is analytic and we can expand it in a Laurent series

$$\mathbf{W}(z) = \sum_n W_n z^{-n-h} \tag{2.11}$$

so that we introduce the modes W_n that acts on the Hilbert space $\mathcal{H}^H$. Then we can obtain the modes W_n through the integration over the curve in the upper half complex plane

$$W_n = \frac{1}{2}\left(\frac{1}{2\pi i}\int_C dz z^{n+h-1} W(z) - \frac{1}{2\pi i}\int_C d\overline{z}\overline{z}^{n+h-1}\Omega(\overline{W})(\overline{z})\right), \tag{2.12}$$

where C is curve in the upper half plane that is labeled as $z = -e^{-i\sigma+\tau}, \sigma \in (0,\pi)$. It is important to stress that there is just one such action of $\mathcal{W}$ constructed out of the two chiral fields $W(z)$ and $\Omega\overline{W}(\overline{z})$.

After this short review of $BCFT$ we return to the SFT and its equation of motion. Note that in the abstract language [1] the open string field theory action (2.2) is

$$S = -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\int \Phi \star Q\Phi + \frac{1}{3}\int \Phi \star \Phi \star \Phi\right) \tag{2.13}$$

from which we immediately get an equation of motion

$$\frac{1}{\alpha'}Q\Phi_0 + \Phi_0 \star \Phi_0 = 0\,. \tag{2.14}$$

It is easy to see that the string field in the form

$$\Phi_0 = e^{-\lambda K_L(\mathcal{I})} \star \frac{1}{\alpha'}Q(e^{\lambda K_L(\mathcal{I})})\,, \qquad \lambda \in \mathrm{R} \tag{2.15}$$

is solution of (2.14) for any ghost number zero operator K_L acting on the SFT star algebra $\star$ identity element $\mathcal{I}$ which is ghost number zero string field that obeys [33]

$$\mathcal{I} \star X = X \star \mathcal{I} = X\,, \tag{2.16}$$

for any string field X.[5] Let us consider operator K in (2.15) from the chiral algebra $\mathcal{W}$ of $BCFT$ in the form

$$K \equiv \mathbf{W} = \frac{1}{2}\left(\frac{1}{2\pi i}\int_C dz W(z) - \frac{1}{2\pi i}\int_C d\overline{z}\Omega(\overline{W})(\overline{z})\right). \tag{2.17}$$

where $W(z), \overline{W}(\overline{z})$ are holomorphic, anti-holomorphic fields respectively of conformal weight $(1,0), (0,1)$ which transform under general conformal transformations $z \to f(z)$ as

$$\begin{aligned} U_f W(z) U_f^{-1} &= \frac{df(z)}{dz} W(f(z)) \equiv f'(z) W(f(z))\,, \\ U_f \overline{W}(\overline{z}) U_f^{-1} &= \frac{d\overline{f}(\overline{z})}{d\overline{z}} \overline{W}(\overline{f}(\overline{z})) \equiv \overline{f}'(\overline{z}) \overline{W}(\overline{f}(\overline{z}))\,. \end{aligned} \tag{2.18}$$

Then we immediately get that $\mathbf{W}$ is invariant under conformal transformations

$$\begin{aligned} f \circ \mathbf{W} \equiv U_f \mathbf{W} U_f^{-1} &= \frac{1}{2\pi i}\int_C dz f'(z) W(f(z)) - \frac{1}{2\pi i}\int_C d\overline{z}\overline{f}'(\overline{z})\overline{W}(\overline{f}(\overline{z})) \\ &= \frac{1}{2\pi i}\int_{f(C)} dz f'(z) W(f(z)) - \frac{1}{2\pi i}\int_{f(C)} d\overline{z}\overline{f}'(\overline{z})\overline{W}(\overline{f}(\overline{z})) = \mathbf{W} \end{aligned} \tag{2.19}$$

using the fact that $\mathbf{W}$ does not depend on the integration contour C as a consequence of gluing conditions (2.9).

We can also define an action of $\mathbf{W}$ on identity field directly in CFT language, following [45, 46, 47]. Instead to taking $|\mathcal{I}\rangle$ as identity element of star algebra we will define its through the relation

$$\langle \mathcal{I} | \,\mathcal{O}\rangle = \langle f_{\mathcal{I}} \circ \mathcal{O}\rangle\,, \qquad |\mathcal{O}\rangle = \mathcal{O}(0)\,|0\rangle\,, \tag{2.20}$$

[5] For recent study of the identity element $\mathcal{I}$, see [42]–[45].

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where

$$f_{\mathcal{I}} = h^{-1}(h(z)^2)\,, \qquad h = \frac{1+iz}{1-iz}\,. \tag{2.21}$$

In this approach the identity field is considered as state that belong to family of wedge states [48, 50, 21]. Wedge state $|n\rangle$ of an angle $\frac{2\pi}{n}$ is defined

$$\langle n\,|\mathcal{O}\rangle = \langle f_n \circ \mathcal{O}\rangle\ , f_n = h^{-1}\left(h(z)^{2/n}\right)\,. \tag{2.22}$$

It follows that $|\mathcal{I}\rangle$ is the wedge state $|n=1\rangle$ of an angle 2π. In this description we define action of the operator $\mathbf{W}$ on $\mathcal{I}$ as

$$\langle \mathcal{I}|\,\mathbf{W}\,|\phi\rangle = \langle f_{\mathcal{I}} \circ \mathbf{W} f_{\mathcal{I}} \circ \phi(0)\rangle\,. \tag{2.23}$$

Using invariance of $\mathbf{W}$ under conformal transformation we immediately get that $\mathbf{W}$ annihilates any wedge state since

$$\begin{aligned}\langle n|\,\mathbf{W}\,|\phi\rangle &= \langle f_n \circ (\mathbf{W}\phi(0))\rangle = \langle \mathbf{W} f_n \circ \phi(0)\rangle \\ &= \left\langle \frac{1}{2\pi i}\oint_C dz \mathbf{W}(z) f_n \circ \phi(0)\right\rangle = 0\end{aligned} \tag{2.24}$$

by deforming the contour C until it shrinks to a point at infinity where there is no other operator. In the upper expression we used standard doubling trick to express $\mathbf{W}$ through holomorphic current $\mathbf{W}(z)$. Let us apply this result for identity field $\mathcal{I}$ and write $\mathbf{W} = \mathbf{W}_L + \mathbf{W}_R$, where subscripts L, R denote the integrals of holomorphic and anti-holomorphic currents $W(z)\,, \overline{W}(\overline{z})$ over left and right side of the string respectively. Then we immediately get

$$\mathbf{W}(\mathcal{I}) = 0 \Rightarrow \mathbf{W}_L(\mathcal{I}) = -\mathbf{W}_R(\mathcal{I})\,. \tag{2.25}$$

For our next purposes it is important following "partial integration formula"

$$\mathbf{W}_L(A) \star B + A \star \mathbf{W}_R(B) = 0\,, \tag{2.26}$$

where $A\,, B$ are general string fields. Recent very nice discussion of the upper expression can be found in [45], where instead of operator $\mathbf{W}$ the BRST operator Q is considered. The proof given there can be easily applied for general operator that is invariant under conformal transformations so that (2.26) is valid for $\mathbf{W}$ too.

Let us return to the solution of SFT (2.15). We observe that it has the form of pure gauge. This fact certainly deserves deeper explanation. As is well known the string field theory action (2.13) is invariant under small gauge transformations

$$\delta\Phi = Q\Lambda - \Lambda \star \Phi + \Phi \star \Lambda\,, \tag{2.27}$$

where Λ is ghost number zero string field. On the other hand the action (2.13) is not generally invariant under the large gauge transformations

$$\Phi' = e^{-\Lambda} \star Q(e^{\Lambda}) + e^{-\Lambda} \star \Phi \star e^{\Lambda}\,. \tag{2.28}$$

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As is well known there is a sharp distinction between the small gauge transformations and the large ones, for very nice discussion, see [58]. As was argued there, small gauge transformation describes redundancy in our description of the theory. On the other hand, large gauge transformations are true symmetries that relate different solutions in given gauge theory which in our case is the open bosonic string field theory. We will see that this interpretation of the large gauge transformation is the appropriate one in case of (2.15). To support this claim let us start to study fluctuation modes around Φ_0. As usually we expand string field Φ as

$$\Phi = \Phi_0 + \Psi \tag{2.29}$$

and insert it in (2.13). Then we obtain an action for the fluctuation field Ψ in the same form as the original one (2.13)

$$S' = -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\int \Psi \star Q'\Psi + \frac{1}{3}\int \Psi \star \Psi \star \Psi\right), \tag{2.30}$$

where the new BRST operator Q' was introduced [33]

$$Q'(X) = Q(X) + \alpha'\Phi_0 \star X - \alpha'(-1)^{|X|} X \star \Phi_0 \,. \tag{2.31}$$

In order to obtain the new form of the BRST operator (2.31) we will follow the calculation given in [28]. We start with the function

$$F(t) = \frac{1}{\alpha'} e^{-\lambda \mathbf{W}_L(\mathcal{I})t} \star Q(e^{\lambda \mathbf{W}_L(\mathcal{I})t}) \,, \qquad F(1) = \Phi_0 \,, \qquad F(0) = 0 \tag{2.32}$$

and perform Taylor expansion around the point $t = 1$

$$\Phi_0 = F(1) = F(0) + \sum_{n=1}^{\infty} \frac{1}{n!}\frac{d^n F}{d^n t}(0) \,, \tag{2.33}$$

where

$$\begin{aligned}
\frac{dF}{dt} &= \frac{\lambda}{\alpha'} e^{-\mathbf{W}_L(\mathcal{I})t} \star [Q, \mathbf{W}]_L(\mathcal{I}) \star e^{\mathbf{W}_L(\mathcal{I})t} \,, \\
\frac{dF}{dt}(0) &= \frac{\lambda}{\alpha'}[Q, \mathbf{W}]_L(\mathcal{I}) \equiv \lambda D_L(\mathcal{I}) \,, \\
\frac{d^2 F}{d^2 t} &= \lambda e^{-\mathbf{W}_L(\mathcal{I})t}(-\mathbf{W}_L(\mathcal{I}) \star D_L(\mathcal{I}) + D_L(\mathcal{I}) \star \mathbf{W}_L(\mathcal{I})) \star e^{\mathbf{W}_L(\mathcal{I})t} \,, \\
\frac{d^2 F}{d^2 t}(0) &= -\lambda \mathbf{W}_L(\mathcal{I}) \star D_L(\mathcal{I}) + \lambda D_L(\mathcal{I}) \star \mathbf{W}_L(\mathcal{I}) = \lambda[\mathbf{W}, D]_L(\mathcal{I}) \,, \\
\frac{d^3 F}{d^3 t}(0) &= \lambda^2[\mathbf{W}_L, [\mathbf{W}_L, D_L]] \,, \ldots \,, \frac{d^n F}{d^n t}(0) = \lambda^n \overbrace{[\mathbf{W}, [\mathbf{W}, \ldots, [Q, \mathbf{W}]]]}^{n-1}{}_L(\mathcal{I})
\end{aligned} \tag{2.34}$$

and consequently

$$\Phi_0 = \frac{1}{\alpha'}\sum_{n=1}^{\infty}\frac{\lambda^n}{n!}\overbrace{[\mathbf{W}, [\mathbf{W}, \ldots, [Q, \mathbf{W}]]]}^{n}{}_L(\mathcal{I}) \equiv \mathcal{D}_L(\mathcal{I}) \,. \tag{2.35}$$

It is important to stress that for validity of the calculation given above $\mathbf{W}$ must obey the relation (2.26). We have also used $Q(\mathcal{I}) = 0 \Rightarrow Q_R(\mathcal{I}) = -Q_L(\mathcal{I}), [Q_R, \mathbf{W}_L] = 0$. ¿From (2.35) see that we can express Φ_0 as a result of the action of the ghost number one operator $\mathcal{D}_L$ acting on the identity field. Then we immediately obtain

$$\begin{aligned} Q'(X) &= Q(X) + \alpha' \mathcal{D}_L(\mathcal{I}) \star X - \alpha'(-1)^{|X|} X \star \mathcal{D}_L(\mathcal{I}) \\ &= Q(X) - \alpha' \mathcal{I} \star \mathcal{D}_R(X) - \alpha' \mathcal{D}_L(X) \star \mathcal{I} = Q(X) - \alpha' \mathcal{D}(X) \\ &= Q(X) + \sum_{n=1}^{\infty} \frac{\lambda^n}{n!} \overbrace{[\mathbf{W}, [\mathbf{W}, \dots, [\mathbf{W}, Q]]]}^{n}(X) = e^{\lambda \mathbf{W}}(Q(e^{-\lambda \mathbf{W}}(X))) . \end{aligned} \tag{2.36}$$

This form of the shifted BRST operator Q' is convenient for the analysis of fluctuation modes around solution (2.15) as we show in the next section.

3. Relation between SFT action S' and the deformation in $BCFT$

In the previous section we have found an exact solution of the string field theory and also an action for fluctuation modes

$$\begin{aligned} S' &= -\frac{1}{g_0^2} \left(\frac{1}{2\alpha'} \int \Psi \star Q' \Psi + \frac{1}{3} \int \Psi \star \Psi \star \Psi \right) \\ &= -\frac{1}{g_0^2} \left(\frac{1}{2\alpha'} \langle I \circ \Psi(0) Q' \Psi(0) \rangle + \frac{1}{3} \langle f_1 \circ \Psi(0) f_2 \circ \Psi(0) f_3 \circ \Psi(0) \rangle \right) , \\ Q'(\Psi(0)) &= e^{\lambda \mathbf{W}} Q e^{-\lambda \mathbf{W}} (\Psi(0)) . \end{aligned} \tag{3.1}$$

Upper expression implies that it is natural to consider following redefinition of fluctuation states

$$|\Psi\rangle = e^{\lambda \mathbf{W}} |\Phi\rangle , \qquad |\Psi\rangle = \Psi(x=0) |0\rangle , \qquad |\Phi\rangle = \Phi(x=0) |0\rangle , \tag{3.2}$$

where $\Psi(x), \Phi(x)$ are boundary operators in $BCFT$ which are localized at point x on the real line. Using the fact that $\mathbf{W}$ annihilates vacuum state $|0\rangle$ we can write

$$\mathbf{W} |\Psi\rangle = \mathbf{W} \Psi(0) |0\rangle = [\mathbf{W}, \Psi(0)] |0\rangle \tag{3.3}$$

hence

$$\begin{aligned} |\Psi\rangle &= e^{\lambda \mathbf{W}} |\Phi\rangle = \sum_{N=0}^{\infty} \frac{\lambda^N}{N!} \overbrace{[\mathbf{W}, [\mathbf{W}, \dots, [\mathbf{W}, \Phi(0)]]]}^{N} |0\rangle , \\ [\mathbf{W}, \Phi(0)] &= \frac{1}{2\pi i} \oint_C dz \mathbf{W}(z) \Phi(0) \end{aligned} \tag{3.4}$$

so that we can define the vertex operator for fluctuation field as

$$\Psi(x) = e^{\mathbf{W}}(\Phi)(x) \equiv \sum_{N=1}^{\infty} \frac{\lambda^N}{2^N N!} \oint_{C_1} \cdots \oint_{C_N} \frac{dz_1}{2\pi i} \cdots \frac{dz_N}{2\pi i} W(z_1) \cdots W(z_N) \Phi(x) , \tag{3.5}$$

where C_i are small circles around the point x and where their radii are given as $\epsilon_{i-1} > \epsilon_i$ and in the end of the calculation we take the limit $\epsilon_i \to 0$. In the previous expression we have slightly moved the insertion point x above to real axis in order to perform contour integration. As a result the second term in $\mathbf{W}$ has no singularity with $\Phi(x)$ and hence we can consider the holomorphic field $W(z)$ only.

When we insert (3.2) into (3.1) we get

$$\begin{aligned} S &= -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\int e^{\lambda\mathbf{W}}(\Phi)\star e^{\lambda\mathbf{W}}Q(e^{-\lambda\mathbf{W}}e^{\lambda\mathbf{W}}(\Phi))+\frac{1}{3}\int e^{\lambda\mathbf{W}}(\Phi)\star e^{\lambda\mathbf{W}}(\Phi)\star e^{\lambda\mathbf{W}}(\Phi)\right)\\ &= -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\left\langle I\circ\left(e^{\lambda\mathbf{W}}(\Phi)(0)\right)e^{\lambda\mathbf{W}}(Q\Phi(0))\right\rangle+\right.\\ &\qquad\left.+\frac{1}{3}\left\langle f_1\circ\left(e^{\lambda\mathbf{W}}(\Phi)(0)\right)f_2\circ\left(e^{\lambda\mathbf{W}}(\Phi)(0)\right)f_3\circ\left(e^{\lambda\mathbf{W}}(\Phi)(0)\right)\right\rangle\right). \end{aligned} \tag{3.6}$$

Using (2.19) we obtain

$$f_i\circ(e^{\lambda\mathbf{W}}(\Phi)(0))=U_{f_i}e^{\lambda\mathbf{W}}U_{f_i}^{-1}\left(U_{f_i}\Phi(0)U_{f_i}^{-1}\right)=e^{\lambda\mathbf{W}}(f_i\circ\Phi(0)) \tag{3.7}$$

so that the SFT action S' for fluctuation modes can be written as

$$\begin{aligned} S' &= -\frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\left\langle e^{\lambda\mathbf{W}}(I\circ\Phi(0))\,e^{\lambda\mathbf{W}}(Q\Phi(0))\right\rangle+\right.\\ &\qquad\left.+\frac{1}{3}\left\langle e^{\lambda\mathbf{W}}(f_1\circ\Phi(0))e^{\lambda\mathbf{W}}(f_2\circ\Phi(0))e^{\lambda\mathbf{W}}(f_3\circ\Phi(0))\right\rangle\right)\\ &= \frac{1}{g_0^2}\left(\frac{1}{2\alpha'}\langle I\circ\Phi(0)Q\Phi(0)\rangle_{W,\lambda}+\frac{1}{3}\langle f_1\circ\Phi(0)f_2\circ\Phi(0)f_3\circ\Phi(0)\rangle_{W,\lambda}\right), \end{aligned} \tag{3.8}$$

where we have defined deformation of boundary correlators [32]

$$\langle\Phi_1(x_1)\dots\Phi_M(x_M)\rangle_{W,\lambda}=\left\langle e^{\lambda\mathbf{W}}(\Phi_1)(x_1)\dots e^{\lambda\mathbf{W}}(\Phi_M)(x_M)\right\rangle. \tag{3.9}$$

The form of the action (3.8) is the main result of our paper which says that when we perform redefinition of fluctuation modes as in (3.5), then the SFT S' (3.1) is the same as the SFT action defined on the background $BCFT''$ that arises from the original one through marginal deformation inserted on the real line $z=\overline{z}$. More precisely, the general prescription of the boundary deformation in given $BCFT$ is as follows. We start with some $BCFT$ with the state space $\mathcal{H}^H_{\Omega,\alpha}$ where (Ω,α) denotes the boundary condition along the real line. Boundary operators $\psi(x)\in\Phi(\mathcal{H}^H)$ may be used to define a new perturbed $BCFT''$ whose correlation functions are constructed from the unperturbed ones by the formal expansion

$$\begin{aligned} \langle\phi_1(z_1,\overline{z}_1)\dots\phi_N(z_N,\overline{z}_N)\rangle_{\alpha,\lambda\psi} &= Z^{-1}\langle I_{\lambda\psi}\phi_1(z_1,\overline{z}_1)\dots\phi_N(z_N,\overline{z}_N)\rangle_\alpha\\ &= Z^{-1}\sum_n\lambda^n\int\dots\int_{x_i<x_{i+1}}\frac{dx_1}{2\pi}\cdots\frac{dx_N}{2\pi}\times\\ &\quad\times\langle\psi(x_1)\dots\psi(x_N)\phi_1\dots\phi_N\rangle_\alpha\,, \end{aligned} \tag{3.10}$$

where λ is a real parameter. From the second line it is clear that the symbol $I_{\lambda\psi}$ in the first line should be understood as a path ordered exponential of the perturbing operator

$$I_{\lambda\psi} = P\exp\left(\lambda S_{\psi}\right) \equiv P\exp\left(\lambda\int_{-\infty}^{\infty}\frac{dx}{2\pi}\psi(x)\right). \tag{3.11}$$

We must mention that given expression is rather formal and suffers from UV divergences and should be regularized. For more detailed discussion, see again [32]. And finally (3.10) defines deformations of bulk correlators only. If there are extra boundary fields present in the correlation function, these formulas have to be modified so that these boundary fields are included in the path ordering. However for special class the boundary deformations these formulas simplify considerably [32]. In (3.10) the boundary operator ψ has conformal dimension h. For $h \neq 1$ the perturbation will automatically introduce length scale and we have to follow the renormalization group flow to come back $BCFT$. However as was stressed in [32] all these general perturbations have common feature that the new $BCFT$ is associated to the same bulk CFT. As a conclusion, the boundary perturbations can only induce changes of the boundary conditions. For marginal deformations with $h = 1$ this implies that the boundary deformation induces the change of the original $BCFT$ with the gluing condition Ω to the new $BCFT''$ with the new gluing condition Ω'', where the precise form of the Ω'' depends on the nature of ψ [32].

Let us consider the deformation of correlators that contain boundary fields as well. It was shown in [32] that (3.10) admits for the obvious generalization

$$\begin{aligned}&\langle\psi_1(u_1)\dots\psi_M(u_M)\phi_1(z_1,\overline{z}_1)\dots\phi_N(z_N,\overline{z}_N)\rangle_{\alpha,\lambda\psi} = \\ &= Z^{-1}\sum_n\frac{\lambda^n}{n!}\int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty}\frac{dx_1}{2\pi}\dots\frac{dx_N}{2\pi}\left\langle\psi(x_1)\dots\psi(x_N)\psi_1\dots\psi_M\phi_1\dots\phi_N\right\rangle_{\alpha},\end{aligned} \tag{3.12}$$

if and only if the boundary fields $\psi_1,\dots,\psi_M$ are local with respect to the perturbing field ψ [32], where two boundary fields $\psi_1(x_1)\,,\psi_2(x_2)$ are said to be local if

$$\psi_1(x_1)\psi_2(x_2) = \psi_2(x_2)\psi_1(x_1)\,, \qquad x_1 < x_2\,. \tag{3.13}$$

This equation is supposed to hold after insertion into arbitrary correlation functions and the right hand side to make sense it is required that there exists a unique analytic continuation from $x_1 < x_2$ to $x_1 > x_2$. We also say that a boundary field $\psi(x)$ is called self local or analytic if it is mutually local with respect to itself. For example, the OPE of a self-local boundary field ψ with conformal dimension $h_\psi = 1$ is determined up to a constant to be

$$\psi(x_1)\psi(x_2) = \frac{K}{(x_1-x_2)^2} + \mathrm{reg}\,, \qquad h_\psi = 1\,. \tag{3.14}$$

After appropriate renormalization [32] the correlation function (3.12) can be written as

$$\begin{aligned}&\langle\psi_1(u_1)\dots\psi_M(u_M)\phi_1(z_1,\overline{z}_1)\dots\phi_N(z_N,\overline{z}_N)\rangle_{\alpha,\lambda\psi} = \\ &= \sum_n\frac{\lambda^n}{n!}\int_{\gamma_1}^{\infty}\dots\int_{\gamma_n}\frac{dx_1}{2\pi}\dots\frac{dx_N}{2\pi}\left\langle\psi(x_1)\dots\psi(x_N)\tilde{\psi}_1\dots\tilde{\psi}_M\phi_1\dots\phi_N\right\rangle_{\alpha},\end{aligned} \tag{3.15}$$

where γ_p is the straight line parallel to the real axis with $\mathrm{Im}\gamma_p = i\epsilon/p$ and where the fields $\tilde{\phi}_i$ are given

$$\tilde{\psi}_i(u_i) = \sum_{n=0}^{\infty} \frac{\lambda^n}{2^n n!} \oint_{C_1} \frac{dx_1}{2\pi} \cdots \oint_{C_n} \frac{dx_n}{2\pi} \psi_i({}_i)\psi(x_n) \ldots \psi(x_1) \,. \tag{3.16}$$

Note that boundary fields $\mathbf{W}$ from the chiral algebra $\mathcal{W}$ are local with respect to themselves to all other boundary and bulk fields in the theory so that (3.15) can be applied to correlators involving arbitrary bulk and boundary fields. Then we see that (3.9) is special case of (3.15) with no bulk operators inserted and that (3.5) has the same form as (3.16). Consequently we can claim that redefinition of fluctuation modes (3.5) in S' (3.6) maps this action to the SFT action (3.8) that describes SFT action defined on background $BCFT''$.

4. Conclusion

In this note we have studied the solution of open bosonic SFT based on the existence of marginal operators $\mathbf{W}$ from the chiral algebra $\mathcal{W}$ of $BCFT$, where $BCFT$ is the classical background on which given SFT is defined. We have mainly focused on the relation between the fluctuation field around the classical solution and the deformed $BCFT''$ that Arieses from the original $BCFT$ by inserting marginal interaction on the real line. We have seen that after an expansion of the string field around the classical solution and its insertion to the original action we obtain the SFT action S' that after redefinition of fluctuation fields is written using $BCFT''$ correlators that are deformations of the correlators in $BCFT$ through introduction of perturbation $\mathbf{W}(x)$ from the chiral algebra $\mathcal{W}$ on the real line. In other words, we have shown that two string field theory actions S_1, S_2 defined using two $BCFT's$, $BCFT_1$, $BCFT_2$ where these two $BCFT's$ are related through marginal deformations from the chiral algebra (It is important that λ is not infinitesimal.), are in fact an expansion of SFT action around different classical solutions. We mean that this result could be considered as an additional evidence of the background independence of open bosonic string field theory [53]–[57] even in case of general deformation parameter λ. We also hope that this result could be helpful for recent application of SFT, for example for the study of the rolling tachyon solution.

However there is am open question that deserves better explanation. The main problem with the exact solution given in this paper is that it resembles pure gauge solution. We have briefly discussed this issue in previous sections but we fell that this problem deserves better explanation. We would like argue as follows. From the study of the marginal deformations in the CFT that was performed in [32] we know that under boundary deformations by fields from the chiral algebra of given $BCFT$ the partition function is invariant and hence the spectrum of the $BCFT$ does not change. The presence of this boundary deformation is mainly seen in the change of the one point bulk function. For example, let us consider open $BCFT$ describing Dp-brane in bosonic theory in flat spacetime. As was shown in [32] the insertion of the boundary perturbation corresponding to the translation in the transverse direction to the world-volume of the D-brane does not change the Hilbert space of given $BCFT$ however it changes one-point function that , roughly speaking, contains the information about the transverse position of D-brane. In other words, from the point of view of the open strings living on D-brane nothing changes. We can interpret this result as a consequence of the translation symmetry of the flat spacetime and hence all transverse positions of D-brane are equivalent. On the other hand we have seen that such a shift of the transverse position of D-brane can be described by exact solution of the open string field theory given in previous section. Since this solution has the form of the pure gauge we could then naively say that it does not describe any new physical situation. However from the equivalence between this solution and the deformation of the $BCFT$ as was shown in this paper and the arguments given above considering marginal deformations in the transverse position of D-brane we mean that this solution really describes new physical situation. On the other hand we have argued above that from the point of view of the open strings living on the world-volume the initial and final positions of D-brane are equivalent and hence it is then natural to expect that this solution could have the form of the pure gauge. On the contrary we expect that solutions of the string field theory equation of motions that correspond to the boundary deformations in the $BCFT$ that do not belong to the chiral algebra of given theory will not have as simple form as a solution given in this paper. We mean that study of these deformations and their relations to the solution of the SFT equation of motion is very interesting problem and we hope to return to it in near future.

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